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Dependence of aspect ratio on magnetic damping of natural convection in low-conducting aqueous solution in a rectangular cavity

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Abstract

Magnetic damping of natural convection in low-conducting aqueous solution was numerically studied for $0 \leqslant$ Hartmann number $(Ha) \leqslant 36.67$, $1000 \leqslant$ Grashof number $(Gr) \leqslant 39810$ and $0.1 \leqslant$ the aspect ratio of a container $(AR) \leqslant 1.0$. The ratio of averaged Nusselt number (Nu_{ave}) with and without magnetic field, $Nu_r = (Nu_{\text{ave}B} - 1)/(Nu_{\text{ave}0} - 1)$, was used to quantify the damping efficiency of natural convection. The results reveal that the magnetic damping of natural convection is strongly dependent on the aspect ratio of the container and there exists an optimum AR for minimizing natural convection. The optimum value of AR is almost independent of Ha and shows the tendency to decrease with increasing Gr. Correlation equations for Nu_r and the ratio of averaged velocity with and without magnetic field with AR, Gr and Ha were given.

Keywords: Aspect ratio; Aqueous solution; Damping of convection; Lorentz force; Magnetic field; Natural convection

1. Introduction

Magnetic damping effects on natural convection in liquid metals have been studied by many researchers since the 1960s, and widely applied to control convection in semiconductor melts such as silicon (see Series and Hurle, 1991; Okada and Ozoe, 1992; Mößner and Müller, 1999). The recent development of a liquid-helium free superconducting magnet has made it easy to use a high magnetic field and control convection in electrically low-conducting fluids, such as the melt of inorganic oxides and aqueous solutions of salts.

One of the most promising applications of damping natural convection by strong magnetic fields is the formation of protein crystals for three-dimensional X-ray structure analysis of protein molecules. When protein crystals are segregated from a protein aqueous solution, protein molecules in the solution are incorporated into the crystal and a low-density region is formed near the crystal surface. Damping of natural convection caused by the density gradient is very important to obtain high quality protein crystals with adequate size and structural quality. For that purpose, microgravity environments in a space shuttle have been used (Lorbe et al., 2000). If we can find other methods for damping convection than in a space shuttle, its influence will be extraordinarily large.

Recently, there have been some reports of the strong magnetic field effects on protein crystallization. According to Yanagiya et al. (2000) and Yin et al. (2000), the growth rate of protein crystals under high magnetic fields was smaller than that without magnetic field. Furthermore, Sato et al. (2000) and Lin et al. (2000) found that the quality of crystals formed under uniform magnetic field of 10 T was superior to the quality of crystals grown in the absence of the field. These results suggest that a strong magnetic field might damp natural convection in aqueous protein solutions, for which the electrical conductivity σ is less than a few tens of Ω^{-1} m^{-1}. For ordinary protein crystal growth experiments, Hartmann number (Ha) is less than 30 even

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under 10 T because of the small size of a container (less than 0.5 cm) and its low σ. Therefore, it is important to know the most efficient experimental conditions to damp natural convection. Under the condition where thermal convection is quenched, natural convection caused by the density gradient near the growing crystal is also damped. Therefore, in this paper, we numerically studied the magnetic damping of thermal convection in a low-conducting aqueous solution as a function of Gr and Ha, focusing on the effects of the aspect ratio of a container, AR.

2. Mathematical model and numerical method

Fig. 1 shows the domain studied in this paper. The fluid in a rectangular container is heated from a vertical wall and cooled from an opposing vertical wall. The other walls are thermally insulated. The direction of an applied magnetic field is along vertical Z-axis. The length of the container along horizontal X-axis, L_x, is the same as the length along the Y-axis, L_y. The length along the Z-axis varies from $0.1L_x$ to $1.0L_x$. The aspect ratio of a container is defined as $AR = L_z/L_x$. The following non-dimensional equations are used to describe fluid flow and heat transfer in our system.

$$\frac{\partial U_i}{\partial X_i} = 0, \tag{1}$$

$$\frac{\partial U_i}{\partial \tau} + U_j\frac{\partial U_i}{\partial X_j} = -\frac{\partial P}{\partial X_i} + \frac{\partial^2 U_i}{\partial X_j \partial X_j} + F_i, \tag{2}$$

$$\frac{\partial \theta}{\partial \tau} + U_i\frac{\partial \theta}{\partial X_i} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial X_i \partial X_i}, \tag{3}$$

$$\frac{\partial^2 \Phi}{\partial X_i \partial X_i} = Pr\left(\frac{\partial U_2}{\partial X_1} - \frac{\partial U_1}{\partial X_2}\right), \tag{4}$$

$$F_i = \left\{ -Ha^2\left(U_1 + \frac{1}{Pr}\frac{\partial \Phi}{\partial X_2}\right), -Ha^2\left(U_2 - \frac{1}{Pr}\frac{\partial \Phi}{\partial X_1}\right), Gr\theta \right\}, \tag{5}$$

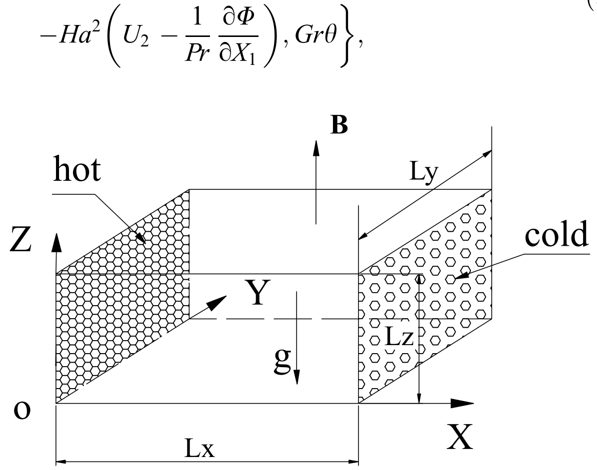

Fig. 1. Schematic configuration studied.

where $i = 1,\ 2,\ 3$. The time, velocity, pressure, and the scalar potential Φ are scaled using reference quantities L_x^2/ν, ν/L_x, $\rho(\nu/L_x)^2$, and αB_z, respectively. B_z is the component of the external magnetic field, α is the thermal diffusivity, and ν is the kinematic viscosity. Hartmann number and Grashof number are defined as $Ha = \sqrt{\sigma B_z^2 L_x^2/\rho\nu}$ and $Gr = g\beta(T_h - T_c)L_x^3/\nu^2$, respectively. Temperature is non-dimensionalized as $\theta = (T - T_c)/(T_h - T_c)$.

The velocity boundary conditions at the walls are the non-slip condition. Boundary conditions for temperature are $\theta = 1$ for the left vertical wall ($X = 0$), and $\theta = 0$ for right vertical wall ($X = 1$). The boundary condition for electric potential is $(\partial\Phi/\partial n)_{\mathrm{wall}} = 0$, where n is the normal direction to the wall. Heat transfer rate at the plate of $X_1 = 0$ is characterized by the surface average Nusselt numbers defined as follows:

$$Nu_{\mathrm{ave}} = \frac{1}{L_y L_z}\int_0^{L_y}\int_0^{L_z}\left(\frac{\partial \theta}{\partial X_1}\right)_{\mathrm{wall}} \mathrm{d}X_2\,\mathrm{d}X_3. \tag{6}$$

Average velocity is also defined as follows:

$$V_{\mathrm{ave}} = \sqrt{\sum_{i=1}^{n_i}\sum_{j=1}^{n_j}\sum_{k=1}^{n_k}|\mathbf{V}|^2 \Big/ n_i n_j n_k}. \tag{7}$$

The governing equations were solved as primitive variables in a uniform, three-dimensional staggered grid based on the finite difference method of Patankar (1980). We validated our code against the experimental results of Okada and Ozoe (1992), and the results agree well with the experimental results.

3. Results and discussion

3.1. Average characteristics

To evaluate the degree of magnetic damping of thermal convection, we define a ratio of average Nusselt number with and without magnetic field as: $Nu_r = (Nu_{\mathrm{ave}B} - 1)/(Nu_{\mathrm{ave}0} - 1)$, where $Nu_{\mathrm{ave}B}$ and $Nu_{\mathrm{ave}0}$ are the average Nusselt numbers in the presence and absence of a magnetic field, respectively.

Fig. 2 shows the dependence of Nu_r on Gr for $0 \leqslant Ha \leqslant 36.67$ and $0.1 \leqslant AR \leqslant 1.0$. When AR is small, the quenching of convection by a magnetic field is nearly independent of Gr. For $AR = 0.1$ and $Ha = 36.67$, Nu_r is about 0.72. On the other hand, for $AR \geqslant 0.5$, Nu_r decreases with decreasing Gr. When $AR = 1.0$, $Ha = 36.67$, and $Gr = 39810$, $Nu_r = 0.81$, and decreases to $Nu_r = 0.16$ for $Gr = 1000$.

Fig. 3 shows the dependence of Nu_r on AR for $1000 \leqslant Gr \leqslant 39810$ and $0 \leqslant Ha \leqslant 36.67$. For each Gr, there is an optimum value of AR for maximizing the damping effect of the magnetic field. The strength of the

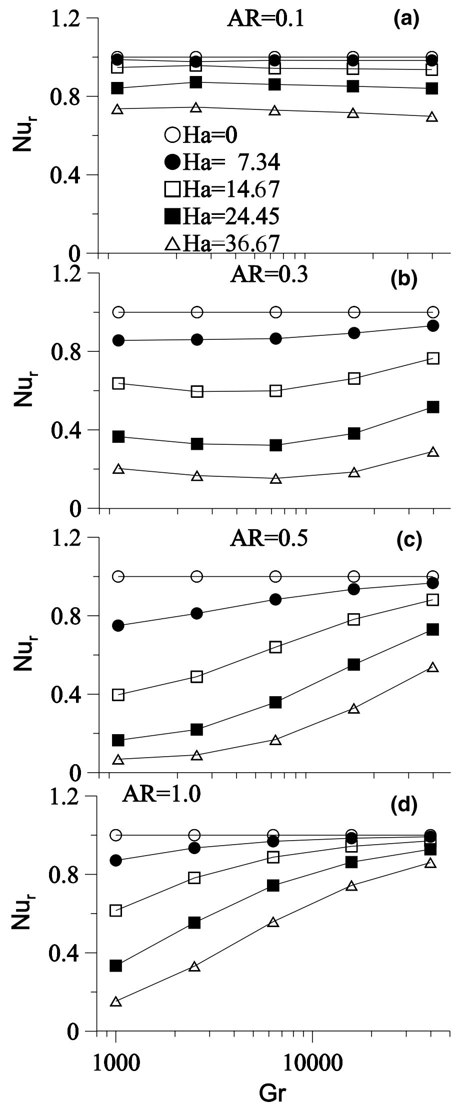

Fig. 2. Dependence of Nu_r on Gr and Ha for various AR.

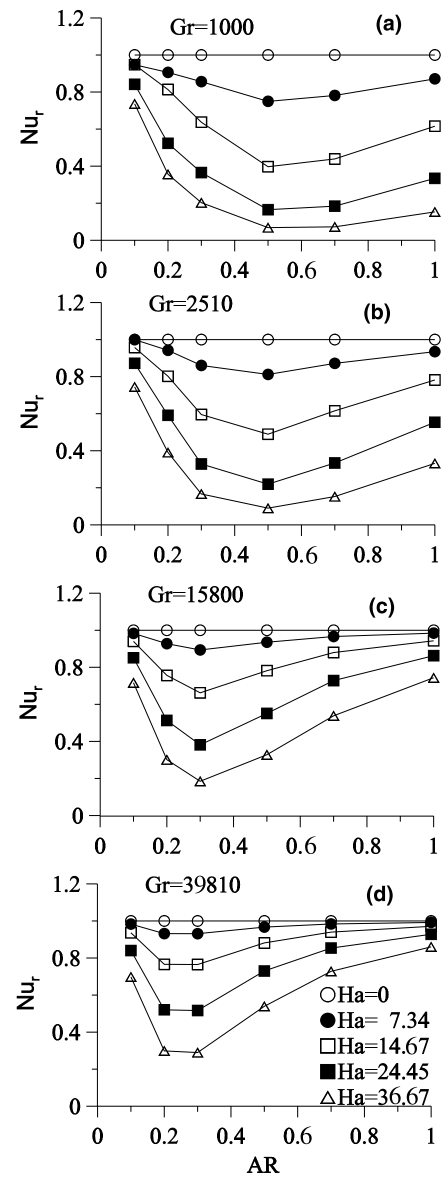

Fig. 3. Dependence of Nu_r on AR and Ha for various Gr.

dependence of Nu_r on AR increases with increasing Gr. For $Gr = 39\,810$ and $Ha = 36.67$, Nu_r achieves a minimum value of 0.28 at $AR = 0.25$. At the same condition $Nu_r = 0.7$ and 0.83 for $AR = 0.1$ and 1.0, respectively. When $Gr = 2510$, the optimum value of AR is about 0.5. The optimum value of AR decreases with increasing Gr, and approaches a relatively small value of $0.2 < AR < 0.3$ when $Gr \geqslant 15\,800$. Furthermore, Fig. 3 indicates that the optimum value of AR is almost independent of Ha.

3.2. Correlation of numerical results

Because there are several parameters such as Gr, Ha, and AR to determine the quenching efficiency, it is important to correlate the numerical results. The correlation result will be useful for designing cells for protein crystal growth and determining the experimental conditions. We also calculated the ratio of average velocity with and without field using Eq. (7) because velocity is an important factor in the process of protein crystal growth. We used $AR^{1.4}Ha/Gr^{0.4AR}$ and $AR^{1.6}Ha/Gr^{AR/3}$ as to correlate Nu_r and the average velocity ratio, respectively. The results are shown in Fig. 4. The best-fit curves are:

$$Nu_r = \frac{Nu_{\mathrm{ave}B} - 1}{Nu_{\mathrm{ave}0} - 1} = 1 - \left[1 + \left(\frac{1.6Gr^{0.4AR}}{AR^{1.4}Ha}\right)^{2.54}\right]^{-0.74}, \tag{8}$$

$$\frac{V_{\mathrm{ave}B}}{V_{\mathrm{ave}0}} = 1 - \left[1 + \left(\frac{1.8Gr^{AR/3}}{AR^{1.6}Ha}\right)^{1.67}\right]^{-1}. \tag{9}$$

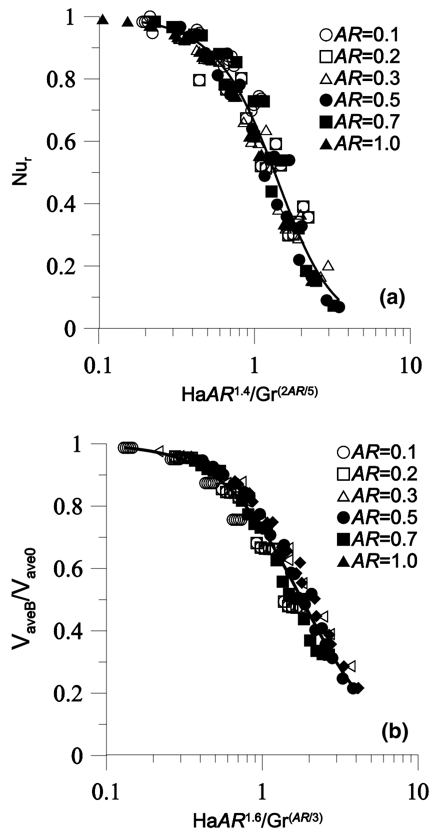

Fig. 4. Correlation of Nu_r (a) and the average velocity ratio (b).

4. Conclusion

Magnetic damping of natural convection in low-conducting aqueous solution in a rectangular cavity was numerically investigated as a function of AR, Gr and Ha. The results reveal that the magnetic damping of natural convection strongly depends on AR and there exists an optimum value for minimizing natural convection. The optimum value of AR is almost independent of Ha and shows the tendency to decrease with increasing Gr.

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